100 Math Questions for Engineering Freshmen

This material is developed to help incoming engineering freshmen in their first Math course. This contains both easy and difficult questions that students may use as they practice solving.

If one finds mistakes in this material, please report it to ceat_ocs.uplb@up,edu.ph.

Q1	Math 27
	Derivatives and Integrals of Trigonometric functions

Question:

Find the derivative of $25\cos\left(sin^2(tanx)\right)$

Solution:

$$\frac{d}{dx}\left(25\cos\left(sin^2(\tan x)\right)\right)$$
$$= \boxed{50\, sec2\,(x)\, cos\,(tan\,x)\, sin\,(tan\,x)\, sin\,(sin2\,(tan\,x))}$$

Q2	Math 27
	Derivatives and Integrals of Trigonometric functions

Question:

Evaluate $\int \frac{\sin x}{cos^2(x)}dx$

Solution:

$$\int \frac{\sin x}{cos^2(x)}dx = \int \frac{\sin(x)}{\cos(x)} \bullet \frac{1}{\cos(x)}dx$$
$$= \int \tan(x)\sec(x)\,dx$$
$$= \boxed{\sec x + c}$$

Q3	Math 27
	Derivatives and Integrals of Trigonometric functions

Question:

Find the derivative of $\frac{2\csc t-1}{\csc t+2}$

Solution 1:

$$\frac{\mathrm{d}}{\mathrm{dx}}\frac{2\csc(t)-1}{csc(t)+2}=\frac{(csc(t)+2)\,(-\csc(t)\cot(t))-(2\csc(t)-1)(-\csc(t)\cot(t))}{(\csc(t)+2)^2}$$

$$=\boxed{\frac{-5\csc(t)\cot(t)}{(\csc(t)+2)^2}}$$

Solution 2:

$$\frac{\mathrm{d}}{\mathrm{dx}}\frac{2\csc(t)-1}{csc(t)+2}=\frac{\mathrm{d}}{\mathrm{dx}}\frac{2\csc(t)-1}{csc(t)+2}$$

$$=\frac{\mathrm{d}}{\mathrm{dx}}\frac{2(-\sin(t))}{1+2\sin(t)}$$

$$=\boxed{\frac{-5\csc(t)\cot(t)}{(\csc(t)+2)^2}}$$

Q4	Math 27
	Derivatives and Integrals of Trigonometric functions

Question:

Evaluate $\int\frac{-2\mathrm{x}(2x^2+1)(6x^3+9x-1)}{(3x^3+1)^3}dx$

Solution:

$$\to\int\frac{-2\mathrm{x}(2x^2+1)(6x^3+9x-1)}{(3x^3+1)^3}dx$$

$$=\int 2\bullet\frac{(2x^2+1)}{(3x^3+1)}\bullet\frac{-x^4-9x^2+4x}{(3x^3+1)^2}dx$$

$$=\int 2\bullet\frac{(2x^2+1)}{(3x^3+1)}\bullet\frac{(3x^3+1)(4x)-(2x^2+1)(9x^2)}{(3x^3+1)^2}dx$$

$$=\boxed{\left(\frac{2x^2+1}{3x^3+1}\right)^{2+C}}$$

Q5	Math 27
	Derivatives and Integrals of Trigonometric functions

Question:

Evaluate $\int \frac{2+7\sin^3 x}{\cos^2 x}$

Solution:

$\rightarrow \int \left(\frac{2}{\cos^2 x} + \frac{7\sin^3 x}{\cos^2 x}\right) dx$

$= \int 2\sec^2 x\, dx + 7\int \frac{\sin^2 x}{\cos^2 x} \sin x\, dx$

$= \int 2\sec^2 x\, dx + 7\int \frac{1-\cos^2 x}{\cos^2 x} \sin x\, dx$

Let $\sin^2 x = 1 \quad \cos^2 x$

$= 2\tan x \quad 7\int \frac{1-u}{u} du$

Let u = $\cos x$
du = -sin$x dx$

$= 2\tan x \quad 7\int u^{-2}\, du\ \int du$

$= 2\tan x \quad 7(\quad u^{-1} \quad u) + C$

$= 2\tan \mathrm{x} + 7\frac{1}{\cos x} + 7\cos x + \mathrm{C}$

$= \boxed{2\tan \mathrm{x} + 7\sec x + 7\cos x + \mathrm{C}}$

Q6	Math 27
	Derivatives and Integrals Yielding Inverse Trigonometric Functions

Question:

Find the derivative of $\sqrt{Arccot\left(\sin\frac{1}{x}\right)}$

Solution:

$= \boxed{\frac{1}{\sqrt{Arccot\left(\sin\frac{1}{x}\right)}}\left(\quad \frac{1}{1+\sin^2\left(\frac{1}{x}\right)}\right)\left(\cos\frac{1}{x}\right)\left(\quad \frac{1}{x^2}\right)}$

Q7	Math 27
	Derivatives and Integrals Yielding Inverse Trigonometric Functions

Question:

Evaluate $\int \frac{sec^2(2x)}{1+9tan^2(2x)}dx$

Solution:

$\rightarrow \int \frac{sec^2(2x)}{1+9tan^2(2x)}dx = \frac{1}{3}\int \frac{du}{1+u^2}$

Let u = 3tan²x
du = 3sec²2x

$= \boxed{\frac{1}{3}\ Arctan(3tan2x)\ +\ C}$

Q8	Math 27
	Derivatives and Integrals Yielding Inverse Trigonometric Functions

Question:

Find the derivative of $Arctan\ \frac{2x}{1-x^2}$

Solution:

$$\frac{d}{dx}Arctan\left(\frac{2x}{1-x^2}\right) = \frac{1}{1+\left(\frac{2x}{1-x^2}\right)^2} \bullet \frac{d}{dx}\left(\frac{2x}{1-x^2}\right)$$

$$= \frac{1}{1+\frac{4x^2}{(1-x)^2}} \bullet \frac{2(1-x^2)-2x(-2x)}{(1-x^2)^2}$$

$$= \frac{2+2x^2}{(1-x^2)+4x^2} = \frac{2(1-x^2)}{(1+x^2)^2}$$

$$= \boxed{\frac{2}{1+x^2}}$$

Q9	Math 27
	Derivatives and Integrals Yielding Inverse Trigonometric Functions

Question:

Evaluate $\int \frac{2}{(x-3)\sqrt{x^2-6x+5}}\,dx$

Solution:

$\rightarrow 2\int \frac{dx}{(x-3)\sqrt{(x-3)^2-4}}$

Let u = x-3
du = dx

$= 2\int \frac{du}{u\sqrt{u^2-2^2}} = 2\left(\frac{1}{2}\right)\sec^{-1}\left(\frac{1}{2}u\right) + C$

$= \boxed{Arcsec\left(\frac{x-3}{2}\right) + C}$

Q10	Math 27
	Derivatives and Integrals Yielding Inverse Trigonometric Functions

Question:

Find the derivative of $\frac{\sin^{-1}x}{1+x}$

Solution:

$\frac{dy}{dx} = \frac{\frac{1+x}{\sqrt{1+x^2}} - \sin^{-1}x}{(1+x)^2}$

$= \boxed{\frac{1+x-\left(\sqrt{1+x^2}\right)\left(\sin^{-1}x\right)}{\left(\sqrt{1+x^2}\right)(1+x)^2}}$

Q11	Math 27
	Derivatives and Integrals Yielding Logarithmic Functions

Question:

Find the derivative of $ln((5x \quad 3)^4(2x^2+7)^3$

Solution:

$$\rightarrow \frac{d}{dx}(ln((5x \quad 3)^4(2x^2+7)^3))$$

$$= \frac{20(5x-3)^3(2x^2+7)^3+12x(5x-3)^4(2x^2+7)^2}{(5x-3)^4(2x^2+7)^3}$$

$$= \boxed{\frac{4(25x^2-9x+35)}{(5x-3)(2x^2+7)}}$$

Q12	Math 27
	Derivatives and Integrals Yielding Logarithmic Functions

Question:

Evaluate $\int \frac{x+6}{x^2+9}dx$

Solution:

$$\rightarrow \int \frac{x+6}{x^2+9}dx \; + \; \int \frac{6dx}{x^2+9}$$

Let u $= x^2 + 9$
du $= 2xdx$

$$= \frac{1}{2}\int \frac{1}{u}du \; + \; (6)\left(\frac{1}{3}\right) Arctan\left(\frac{x}{3}\right) + C$$

$$= \left(\frac{1}{2}\right) ln|x^2+9| \; + \; 2Arctan\left(\frac{x}{3}\right) + C$$

$$= \boxed{\frac{ln|x^2+9|}{2} + \; 2Arctan\left(\frac{x}{3}\right) + C}$$

Q13	Math 27
	Derivatives and Integrals Yielding Logarithmic Functions

Question:

Find the derivative of $y = x^{x^2}$

Solution:

Let $y = x^{x^2}$

$$ln\, y = ln\, x^{x^2}$$

$$ln\, y = x^2\, ln\, x$$

$$\frac{d}{dx}(lny) = \frac{d}{dx}(x^2 lnx)$$

$$\frac{1}{y}\frac{d}{dx} = x^2\frac{1}{x} + (2xlnx)$$

$$\frac{dy}{dx} = y\,[x + 2xlnx]$$

$$\boxed{\frac{dy}{dx} = x^{x^2+1}(1 + 2xlnx)}$$

Q14	Math 27
	Derivatives and Integrals Yielding Logarithmic Functions

Question:

Evaluate $\int \csc x\, dx$

Solution:

$$\rightarrow \int \csc x\, dx = \int \csc x \frac{\csc x + \cot x}{\csc x + \cot x} dx$$

$$= \int \frac{(csc^3 x + \csc x\ \cot x)\, dx}{\csc x + \cot x}$$

$$= \int \frac{-du}{u} = \ln|u| + C$$

$$= \boxed{\ln|\csc x + \cot x| + C}$$

Q15	Math 27
	Logarithmic Differentiation

Question:

Find the derivative of $y = \frac{(5x-2)^6(x^2-1)^4}{\sqrt{tanx}}$

Solution:

Q16	Math 27
	Logarithmic Differentiation

Question:

Find the derivative of $y = x^{\cos(2x)}, x > 0$

Solution:

$$\rightarrow \frac{d}{dx}\left(x^{\cos(2x)}\right) : \ |y| \ = \ \left|x^{\cos(2x)}\right|$$

$$\ln|y| = \cos 2x \ln|x|$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^{\cos(2x)}} \cos 2x \, x^{\cos 2x - 1}$$

$$\frac{dy}{dx} = x^{\cos(2x)}\left(\frac{\cos 2x \, x^{\cos 2x-1}}{x^{\cos(2x)}}\right)$$

$$\boxed{\frac{dy}{dx} = x^{\cos(2x)}\left(\frac{\cos 2x}{x} \quad 2\ln|x|(\sin 2x)\right)}$$

Q17	Math 27
	Logarithmic Differentiation

Question:

Find the derivative of $y = \frac{\sqrt{1-x^2}}{(x+1)^{2/3}}$

Solution:

$$\rightarrow y = \frac{\sqrt{1 \quad x^2}}{(x+1)^{\frac{2}{3}}} = \frac{(1+x)^{1/2}(1 \quad x)^{1/2}}{(x+1)^{2/3}}$$

$$\rightarrow y = (1 \quad x)^{1/2}(1+x)^{-1/6}$$

$$\ln y = \frac{1}{2}\ln(1 \quad x) \quad \frac{1}{6}\ln(1+x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2} \bullet \frac{1}{1 \quad x} \quad \frac{1}{6} \bullet \frac{1}{1+x}$$

$$\frac{dy}{dx} = y \bullet \frac{3(1+x) \quad (1 \quad x)}{6(1 \quad x)(1+x)}$$

$$\boxed{\frac{dy}{dx} = (1 \quad x)^{1/2}(1+x)^{-1/6}}$$

Q18	Math 27
	Logarithmic Differentiation

Question:

Find the derivative of $y = \frac{x^3+2x}{\sqrt[5]{x^7+1}}$

Solution:

$$\rightarrow y = \frac{x^3+2x}{\sqrt[5]{x^7+1}}; \ |y| = \frac{|x^3+2x|}{|x^7+1|^{1/5}}$$

$$\ln|y| = \ln|x^3+2x| \quad \frac{1}{5}\ln|x^7+1|$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3x^2+2}{x^3+2x} \quad \frac{7x^6}{5(x^7+1)}$$

$$\boxed{\frac{dy}{dx} = \frac{x^3+2x}{(x^7+1)^{1/5}}\left[\frac{3x^2+2}{x^3+2x} \quad \frac{7x^6}{5(x^7+1)}\right]}$$

Q19	Math 27
	Logarithmic Differentiation

Question:

Differentiate y: $y = \frac{\sqrt{5x-8}\,\sqrt[3]{1-9cos(4x)}}{\sqrt[4]{x^2+10x}}$

Solution:

$$ln[f(x)] = ln\left|\sqrt{5x\quad 8}\,\sqrt[3]{1\quad 9cos(4x)}\right| \quad ln\left|\sqrt[4]{x^2+10x}\right|$$

$$= \frac{1}{2}ln|5x+8| + \frac{1}{3}ln|1\quad 9cos(4x)| \quad \frac{1}{4}ln|x^2+10x|$$

$$\frac{f'(x)}{f(x)} = \frac{1}{2}\,\frac{5}{5x+8} + \frac{1}{3}\,\frac{36sin4x}{1-9cos4x} \quad \frac{1}{4}\,\frac{2x+10}{x^2+10x}$$

$$f'(x) = f(x)\left[\frac{5/2}{5x+8} + \frac{12sin4x}{1-9cos4x} \quad \frac{\frac{1}{2}x+5/2}{x^2+10x}\right]$$

$$f'(x) = \boxed{\frac{\sqrt{5x-8}\,\sqrt[3]{1-9cos(4x)}}{\sqrt[4]{x^2+10x}}\left[\frac{5/2}{5x+8} + \frac{12sin4x}{1-9cos4x} \quad \frac{\frac{1}{2}x+5/2}{x^2+10x}\right]}$$

Q20	Math 27
	Derivatives and Integrals of Exponential Functions

Question:

Evaluate $\int \frac{e^{Arcsec\,x}}{x\sqrt{x^2-1}}dx$

Solution:

$$\int \frac{e^{Arcsec\,x}}{x\sqrt{x^2-1}}dx = \int e^u$$

$$\boxed{= e^{Arcsec\,x} + C}$$

Let u = Arcsec x

du = $\frac{1}{x\sqrt{x^2-1}}dx$

Q21	Math 27
	Derivatives and Integrals of Exponential Functions

Question:

Evaluate $\int \frac{2^{lnx}}{x(2^{lnx}-1)}dx$

Solution:

$$\int \frac{2^{lnx}}{x(2^{lnx}-1)}dx = \frac{1}{\ln(2)}\int \frac{1}{u}du$$

Let u = $2^{\ln x} \quad 1$
du = $\frac{x}{\ln(2)\bullet 2^{\ln x}}dx$

$$= \frac{\ln(u)}{\ln(2)}$$

$$\boxed{= \frac{\ln(2^{\ln x}-1)}{\ln(2)} + C}$$

Q22	Math 27
	Derivatives and Integrals of Exponential Functions

Question:

Find the derivative of $y = \frac{e^x-e^{-x}}{e^x+e^{-x}}$

Solution:

$$= \frac{d}{dx}\left(\frac{e^x \quad e^{-x}}{e^x + e^{-x}}\right)$$

$$= \frac{(e^x + e^{-x})(e^x + e^{-x}) \quad (e^x \quad e^{-x})(e^x \quad e^{-x})}{(e^x + e^{-x})^2}$$

$$= \frac{(e^x + e^{-x})^2 \quad (e^x \quad e^{-x})^2}{(e^x + e^{-x})^2}$$

$$= \frac{(e^{2x} + 2 + e^{-2x}) \quad (e^{2x} \quad 2 + e^{-2x})}{(e^x + e^{-x})^2}$$

$$\boxed{= \frac{4}{(e^x + e^{-x})^2}}$$

Q23	Math 27
	Derivatives and Integrals of Exponential Functions

Question:

Evaluate $\int \frac{1}{1+e^x} dx$

Solution:

$\rightarrow \int \frac{1}{1+e^x} dx = \int \frac{e^{-x}}{e^{-x}+1} dx$

Let u $= e^{-x} + 1$
du $= e^{-x} dx$

$\int \frac{-du}{u}$

$= \ln|u| + C$

$\boxed{= \ln|e^{-x} + 1| + C}$

Q24	Math 27
	Derivatives and Integrals of Exponential Functions

Question:

Find $\frac{dy}{dx} : y = \frac{x}{1-e^x}$

Solution:

$f'(x) = \frac{(1)(1-e^x) - x(-e^x)}{(1-e^x)^2}$

$f'(x) = \boxed{\frac{1-e^x+xe^x}{(1-e^x)^2}}$

Q25	Math 27
	Applications: Maxima-Minima, Related Rates, Laws of Natural Growth and Decay

Question:

Let the bottom of the embankment (O) be x feet from the bottom of the ladder and y feet from the top of the ladder. As the bottom of the ladder moves toward the embankment, determine the rate at which the top of the ladder is moving.

Solution:

$$\textit{Law of cosines: } x^2 + y^2 \quad 2xy\cos 120^o = 400$$

$$x^2 + y^2 \quad 2xy\left(\quad \frac{1}{2}\right) = 400$$

$$y^2 + xy + (x^2 \quad 400) = 0$$

$$y = \frac{x + \sqrt{x^2 \quad 4(x^2 \quad 400)}}{2}$$

$$y = \quad \frac{x}{2} + \frac{\sqrt{1600 \quad 3x^2}}{2}$$

$$\frac{dy}{dx} = \frac{1}{2} \quad \frac{3}{2\sqrt{1600 \quad 3x^2}}$$

$$\frac{dy}{dx} = \left(\frac{1}{2} \quad \frac{3}{2\sqrt{1600 \quad 3x^2}}\right)\frac{dx}{dt}$$

Since $\frac{dx}{dt}$ *= -1 when x = 4,*

$$\left.\frac{dx}{dt}\right|_{x=4} = \left(\quad \frac{1}{2} \quad \frac{3}{2\sqrt{1600 \quad 3(4)^2}}\right)(\quad 1)\frac{dx}{dt}$$

$$\approx 0.6523\ {}^{ft}/_{s}$$

$\therefore$ *t e ladder is moving at t e rate of* $0.6523\ {}^{ft}/_{s}$ *at t e given instant*

Q26	Math 27
	Applications: Maxima-Minima, Related Rates, Laws of Natural Growth and Decay

Question:

Find the absolute minimum and maximum value of the graph:

$$f(x) = \begin{cases} 2x \quad 7, & 1 \leq \quad x \leq 2 \\ \quad 1 \quad x^2, & 2 \leq x \leq 4 \end{cases}$$

Solution:

$$f(x) = \begin{cases} 2x \quad 7, & 1 \leq \quad x \leq 2 \\ 1 \quad x^2, & 2 \leq x \leq 4 \end{cases} ;\ I = [\quad 1, 4]$$

$$f'(x) = \begin{cases} 2 \quad , & 1 \leq \quad x \leq 2 \\ \quad 2x\ , & 2 \leq x \leq 4 \end{cases}$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(2x \quad 7) = \quad 3$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(1 \quad x^2) = \quad 3$$

*Because f is continuous on I, f has an absolute maximum and minimum value on I.

Since $f\,'(2)$ does not exist, 2 is the critical number of *f.*

$\rightarrow f(\quad 1) = \quad 9$

$\rightarrow f(2) = \quad 3$

$\rightarrow f(4) = \quad 15$

$\therefore$ *Maximum is* $\quad 3$ *and minimum is* $\quad 15$

Q27	Math 27
	Applications: Maxima-Minima, Related Rates, Laws of Natural Growth and Decay

Question:

A horizontal trough is 16 meters long, and its ends are isosceles trapezoids with an altitude of 4 m, a lower base of 4 m, and an upper base of 6m. If the water is being poured into the trough at the rate of 10m3/min. How fast is the water level rising when the water is 2m deep?

Solution:

$$V = \frac{1}{2}(4 + x)(y)(16) \rightarrow V = 8y(x + 4)$$

Express x as a function of y using similar triangles,

Let x = width of the surface of the water

y = depth of the water

$$\rightarrow \frac{x \quad 4}{y} = \frac{2}{4}$$

$$\rightarrow V = 8y\left(\frac{y}{2} + 8\right) = 4y^2 + 64y$$

$$\rightarrow \frac{dV}{dt} = (8y + 64)\frac{dy}{dt}$$

Substitute $\frac{dV}{dt} = 10$ and $y = 2$,

$$\rightarrow 10 = 80\frac{dy}{dt}$$

$$\boxed{\frac{dy}{dt} = \frac{1}{8} {}^{m}/_{min}}$$

Q28	Math 27
	Applications: Maxima-Minima, Related Rates, Laws of Natural Growth and Decay

Question:

The population of rats in an area is 50 and after a year there are already 800 rats. When will the population reach 30000?
(Assume 1. No extermination/ death of rats, 2. Exponential growth)

Solution:

$$8000 = 500e^{3k}[P = P_0e^{tk}]$$

Finding k:

$$\frac{80}{5} = e^{3k} \rightarrow ln\left(\frac{80}{5}\right) = 3k$$

$$\rightarrow k = \frac{ln(16)}{3}$$

Finding t, given P = 30000

$$30000 = 500e^{t\frac{ln(16)}{3}}$$

$$60 = e^{t\frac{ln(16)}{3}}$$

$$\rightarrow ln\left(\frac{60}{1}\right) = t\left[\frac{ln(16)}{3}\right]$$

$t = \frac{3ln(60)}{ln(16)}$ years.

Q29	Math 27
	Applications: Maxima-Minima, Related Rates, Laws of Natural Growth and Decay

Question:

Find a function Q(t) that gives the number of bacteria on a petri dish given that the rate of growth of bacteria is q(t) = 3t , where t is in hours and q(t) is in thousands of bacteria per hour. The culture starts with 10000 bacteria.

Solution:

$$Q(t) = \int q(t)dt$$

$$= \int 3^t dt$$

$$= \frac{3^t}{ln3} + C$$

$$\rightarrow @\, t = 0, Q(0) = 10 = \frac{1}{ln3} + C$$

$$\rightarrow C = 10 \quad \frac{1}{ln3}$$

$$\rightarrow \therefore \boxed{Q(t) = \frac{3t}{ln3} + 10 \quad \frac{1}{ln3}}$$

Q30	Math 27
	Integration by Parts

Question:

Evaluate $\int (x+1)^2 \ln(3x)\, dx$

Solution:

let $u = ln3x$

$du = \frac{1}{x} dx$

$dv = (x+1)^2 dx$

$v = \frac{(x+1)^3}{3}$

$$\int (x+1)^2 \ln(3x)\, dx = uv \quad \int vdu$$

$$= \frac{1}{3}(x+1)^3 \quad \frac{1}{3}\int \frac{(x+1)^3}{x} dx$$

$$= \frac{1}{3}(x+1)^3 \ln 3x \quad \frac{1}{3}\int \frac{x^3+3x^2+3x+1}{x} dx$$

$$= \frac{1}{3}(x+1)^3 \ln 3x \quad \frac{1}{3}\int \left(x^2 + 3x + 3 + \frac{1}{x}\right) dx$$

$$= \boxed{\frac{1}{3}(x+1)^3 \ln 3x \quad \frac{1}{3}\left[\frac{x^3}{3} + \frac{3x^2}{2} + 3x + \ln x\right] + C}$$

Q31

Math 27

Integration by Parts

Question:

Evaluate $\int x^3 e^{x^2}\, dx$

Solution:

Let $\mathrm{u} = x^2$
$\mathrm{du} = 2x dx$

$$\int x^3 e^{x^2}\, dx = \int \frac{x^3 e^u}{2x} du$$

$$= \frac{1}{2}\int x^2 e^u du = \frac{1}{2}\int u e^u du$$

let $v = u \quad dw = e^u du$
$dv = du \quad w = e^u$

$$= \frac{1}{2}(vw \quad \int w dv)$$

$$= \frac{1}{2}(vw \quad \int w dv)$$

$$= \frac{1}{2}(ue^u \quad \int e^u du)$$

$$= \frac{1}{2}(ue^u \quad e^u) + C$$

$$= \boxed{\frac{1}{2}\left(x^2 e^{x^2} \quad e^{x^2}\right) + \mathrm{C}}$$

Q32

Math 27

Integration by Parts

Question:

Evaluate $\int sec^3 x\, dx$

Solution:

let $u = sec\, x$
$du = sec\, x\, tan\, x$
$dv = sec^2 x\, dx$
$v = tan\, x$

$$= sec\, x\, tan\, x \quad \int tan\, x\, sec\, x\, tan\, x$$

$$tan^2 x = sec^2 x \quad 1$$

$$= secx\, tanx \quad \int (sec^3 x \quad secx)dx$$

$$= \int secxdx = secx\, tanx + \ln|secx + tanx| \quad \int sec^3 xdx$$

$$= 2\int secxdx = secx\, tanx + \ln|secx + tanx|$$

$$= \int secxdx = \boxed{\frac{secx\, tanx + \ln|secx + tanx|}{2}}$$

Q33	Math 27
	Integration by Parts

<u>Question:</u>

Evaluate $\int Arctan\,\sqrt{x}\,dx$

<u>Solution:</u>

$$\rightarrow \frac{d}{dx}\left(x^{\cos(2x)}\right) : \ |y| \ = \ \left|x^{\cos(2x)}\right|$$

$$\ln|y| = \cos 2x \ln|x|$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^{\cos(2x)}} \cos 2x\, x^{\cos 2x - 1}$$

$$\frac{dy}{dx} = x^{\cos(2x)} \left(\frac{\cos 2x\, x^{\cos 2x-1}}{x^{\cos(2x)}}\right)$$

$$\boxed{\frac{dy}{dx} = x^{\cos(2x)} \left(\frac{\cos 2x}{x} \quad 2\ln|x|(\sin 2x)\right)}$$

Q34	Math 27
	Integration by Parts

<u>Question:</u>

Evaluate $\int (4x^3 \quad 9x^2 + 7x + 3)e^{-x}\,dx$

<u>Solution:</u>

Using successive IBP:

$$= (4x^3 \quad 9x^2 + 7x + 3)(\ e^{-x}) \quad (12x^2 \quad 18x + 7)(e^{-x})$$
$$+(24x \quad 18)(\ e^{-x}) \quad 24(e^{-x}) + C$$

$$= \quad e^{-x}(4x^3 \quad 9x^2 + 7x + 3) \quad e^{-x}(12x^2 \quad 18x + 7)$$
$$e^{-x}(24x \quad 18) \quad 24e^{-x} + C$$

$$\boxed{= \quad e^{-x}(4x^3 + 3x^2 + 13x + 16) + C}$$

Q35	Math 27
	Trigonometric Substitution

Question:

Evaluate $\int \frac{1}{(x^2+25)^{\frac{3}{2}}}dx$

Solution:

$$= \int \frac{5sec^2\theta d\theta}{(5sec\theta)^3}$$

Let $\quad x = 5tan\theta$ $dx = 5sec^2\theta d\theta$ $\sqrt{x^2+25} = 5sec\theta$

$$= \int \frac{5sec^2\theta}{125sec^3\theta}d\theta$$

$$= \frac{1}{25}\int \frac{1}{sec\theta}d\theta$$

$$= \frac{1}{25}\int cos\theta d\theta = \frac{sin\theta}{25} + C = \frac{1}{25}\left(\frac{tan\theta}{sec\theta}\right) + C$$

$$= \boxed{\frac{x}{25\sqrt{x^2+25}} + C}$$

Q36	Math 27
	Trigonometric Substitution

Question:

Evaluate $\int \frac{1}{x^2\sqrt{4x^2-9}}dx$

Solution:

$$= \frac{\frac{3}{2}sec\theta tan\theta d\theta}{\frac{9}{4}sec^2\theta(3tan\theta)}$$

Let $\quad 2x = 3\ sec\theta$ $2dx = 3\ sec\theta\ tan\theta\ d\theta$ $\sqrt{4x^2\ \ 9} = 3\ tan\theta$

$$= \int \frac{2}{9sec\theta}d\theta$$

$$= \frac{2}{9}(sin\theta) + C$$

$$= \boxed{\frac{2}{9}\left(\frac{\sqrt{4x^2-9}}{2x}\right) + C}$$

Q37	Math 27
	Trigonometric Substitution

Question:

Evaluate $\int \frac{\ln^3 x}{x\sqrt{ln^2\,(x)-4}}dx$

Solution:

$$= \int \frac{u^3 du}{\sqrt{u^2-4}}$$

$$= \int \frac{8\sec^3\theta(2sec\theta tan\theta d\theta)}{2tan\theta}$$

$$= 8\int \sec^4\theta d\theta$$

$$= 8\int(\tan^2\theta + 1)\sec^2\theta d\theta$$

$$= 8\int \tan^2\theta(\sec^2\theta)d\theta + 8\int \sec^2\theta d\theta$$

$$= \frac{8}{3}\left[\frac{(u^2-4)^{3/2}}{8}\right] + 8\left[\frac{\sqrt{u^2-4}}{2}\right] + C$$

$$= \boxed{\frac{8}{3}\left[\frac{(ln^2x-4)^{3/2}}{8}\right] + 8\left[\frac{\sqrt{ln^2x-4}}{2}\right] + C}$$

Let $u = \ln x$
$du = \frac{dx}{x}$

Trigo Sub
Let $u = 2\,sec\theta$
$du = 2\,sec\theta\,tan\theta\,d\theta$
$\sqrt{u^2 \quad 4} = 2\,tan\theta$

Q38	Math 27
	Trigonometric Substitution

Question:

Evaluate $\int \frac{x^2}{\sqrt{x^2+6}}dx$

Solution:

$$= \int \frac{(6\tan^2\theta)\sqrt{6}\sec^2\theta d\theta}{\sqrt{6}sec\theta}$$

$$= 6\int \tan^2\theta\, sec\,\theta\, d\theta$$

$$= 6\int(\sec^2\theta \quad 1)sec\,\theta\, d\theta$$

$$= 6[\int \sec^3\theta\, d\theta \quad \int sec\,\theta\, d\theta]$$

$$= 6[(sec\theta tan\theta \int \tan^2\theta\, sec\,\theta\, d\theta) \quad \int sec\theta d\theta]$$

$$= 6\left[\frac{1}{2}\,sec\theta\,tan\theta + \frac{1}{2}ln|sec\theta\,tan\theta| \quad \int sec\theta d\theta\right]$$

$$= 6\left[\frac{1}{2}\,sec\theta\,tan\theta \quad \frac{1}{2}ln|sec\theta\,tan\theta|\right] + C$$

$$= \boxed{\frac{3\sqrt{x^2+6}}{\sqrt{6}}\left(\frac{x}{\sqrt{6}}\right) \quad 3\ln\left|\frac{\sqrt{x^2+6}}{\sqrt{6}} + \frac{x}{\sqrt{6}}\right| + C}$$

Trigo Sub
Let $\theta = Arctan\frac{x}{\sqrt{6}}$
$x = \sqrt{6}tan\theta$
$dx = \sqrt{6}\sec^2\theta d\theta$
$\sqrt{x^2+6} = \sqrt{6\tan^2\theta + 6}$
$= \sqrt{6(\tan^2\theta + 1)}$
$= \sqrt{6}\sec\theta$

for $\int \sec^3 x$: use IBP
Let $u = secx$
$du = secxtanxdx$
$dv = \sec^2 xdx$
$v = \tan x$

for $\int \sec^3\theta\, d\theta$:
$\int \sec^3\theta\, d\theta = sec\theta\,tan\theta \quad \int(\sec^3\theta \quad sec\theta)\,d\theta$
$= sec\theta\,tan\theta + \ln|sec\theta\,tan\theta| + C$
$\int \sec^3\theta\, d\theta$
$2\int \sec^3\theta\, d\theta = sec\theta\,tan\theta + \ln|sec\theta\,tan\theta| + C$
$\therefore \int \sec^3\theta\, d\theta = \frac{1}{2}\,sec\theta\,tan\theta + \frac{1}{2}\ln|sec\theta\,tan\theta| + C$

Q39	Math 27
	Trigonometric Substitution

Question:

Evaluate $\int \cos(x)\sqrt{9+25sin^2(x)}\,dx$

Solution:

$$\rightarrow \int cos(x)\sqrt{9+25\left[\frac{3}{5}tan(\theta)\right]^2}\,dx$$

$$= \int \frac{3}{5}\sec^2\theta\sqrt{9+25\left[\frac{3}{5}tan(\theta)\right]^2}\,d\theta$$

$$= \int \frac{3}{5}\sec^2\theta\sqrt{9+9\tan^2\theta}\,d\theta$$

$$= \int \frac{3}{5}\sec^2(\theta)\,3\sqrt{sec^2\theta}\,d\theta$$

$$= \frac{9}{5}\int \sec^3(\theta)\,d\theta$$

$$= \frac{9}{10}\left[sec(\theta)tan(\theta) + ln|sec(\theta)+tan(\theta)|\right] + C$$

$$= \boxed{\frac{sin(x)\sqrt{9+25\sin^2(x)}}{2} + \frac{9}{10}ln\left|\frac{sin(x)\sqrt{9+25\sin^2(x)}}{3}\right| + C}$$

Let
$sin(x) = \frac{3}{5}tan(\theta)$
$cos(x)dx = \frac{3}{5}sec(\theta)d\theta$
$tan\theta = \frac{5sin(x)}{3}$
$sec\theta = \frac{\sqrt{3^2+(5sinx)^2}}{3}$

Q40	Math 27
	Integration by Partial Fractions

Question:

Evaluate $\int \frac{4x+3}{2x^2+3x+1}dx$

Solution:

$$\int \frac{4x+3}{2x^2+3x+1}dx;\ \frac{P}{Q} = \frac{4x+3}{2x^2+3x+1} = \frac{4x+3}{(2x+1)(x+1)}$$

$$= \frac{A}{x+1} + \frac{B}{2x+1}$$

$$= \frac{A(2x+1)+B(x+1)}{(x+1)(2x+1)}$$

$$\rightarrow 4x+3 = A(2x+1) + B(x+1)$$

$$\rightarrow if\ x = \frac{1}{2}, B = 2$$

$$\rightarrow if\ x = \quad 1, A = 1$$

$$\int \frac{4x+3}{2x^2+3x+1}dx = \int \frac{1}{x+1}dx + \int \frac{2}{2x+1}dx$$

$$= \boxed{ln|x+1| + 2ln|2x+1| + C}$$

Q41	Math 27
	Integration by Partial Fractions

Question:

Evaluate $\int \frac{3x+1}{x^3+2x^2+x} dx$

Solution:

$\int \frac{3x+1}{x^3+2x^2+x} dx; \ \frac{P}{Q} = \frac{3x+1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$= \frac{A(x+1)^2+B(x)(x+1)+C(x)}{x(x+1)^2}$

$\rightarrow 3x + 1 = A(x+1)^2 + Bx(x+1) + Cx$

$\rightarrow if \ x = \quad 1, C = 2$

$\rightarrow if \ x = 0, A = 1, t \quad en \ B = \quad 1$

$\int \frac{3x+1}{x^3+2x^2+x} dx = \int \left(\frac{1}{x} \quad \frac{1}{x+1} + \frac{2}{(x+1)^2}\right) dx$

$= \boxed{ln|x| \quad ln|x+1| \quad \frac{2}{x+1} + C}$

Q42	Math 27
	Integration by Partial Fractions

Question:

Evaluate $\int \frac{x+3}{(x-1)^3} dx$

Solution:

Rewrite the fraction:

$\rightarrow \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} = \frac{x+3}{(x-1)^3} = \frac{A(x-1)^2}{(x-1)^2} + \frac{B(x-1)}{(x-1)^3} + \frac{C}{(x-1)^3}$

$= \frac{Ax^2-2Ax+A+Bx-B+C}{(x-1)^3}$

$\frac{x+3}{(x-1)^3} = \frac{Ax^2-2Ax+A+Bx-B+C}{(x-1)^3}$

$$\begin{aligned} &\rightarrow A &&= 0 \qquad &&\therefore A = 0 \\ &\quad 2A + B &&= 1 &&B = 1 \\ &A \quad B + C &&= 3 &&C = 4 \end{aligned}$$

$\int \frac{x+3}{(x-1)^3} dx = \int \left[\frac{1}{(x-1)^2} + \frac{4}{(x-1)^3}\right] dx$

$= \int \frac{1}{(x-1)^2} dx + \int \frac{4}{(x-1)^3} dx$

$= \boxed{\frac{-1}{x-1} \quad \frac{2}{(x-1)^2} + C}$

Q43	Math 27
	Integration by Partial Fractions

Question:

Evaluate $\int \frac{2x-1}{(x-5)^2} dx$

Solution:

Rewrite the fraction:

$$\rightarrow \frac{A}{x-5} + \frac{B}{(x-5)^2} = \frac{2x-1}{(x-5)^2} = \frac{A(x-5)}{(x-5)^2} + \frac{B}{(x-5)^2} = \frac{Ax+5A+B}{(x-5)^2}$$

$$\rightarrow 2x \quad 1 \ = Ax \quad 5A + B$$

$$\rightarrow A \qquad = 2 \qquad \therefore A = 2$$
$$5A + B = \quad 1 \qquad C = 4$$

$$\rightarrow \int \frac{2x-1}{(x-5)^3} dx = \int \frac{2}{x-5} dx + \int \frac{9}{(x-5)^2} dx$$

$$= \boxed{2ln|x \quad 5| \quad \frac{9}{x-5} + C}$$

Q44	Math 27
	Integration by Partial Fractions

Question:

Evaluate $\int \frac{2+x^4}{x^3+9x} dx$

Solution:

Rewrite the fraction:

$$\int \frac{2+x^4}{x^3+9x} dx \rightarrow \frac{2+x^4}{x^3+9x} = x + \frac{2 \quad 9x^2}{x(x^2+9)}$$

$$\rightarrow \frac{2-9x^2}{x(x^2+9)} = \frac{A}{x} + \frac{Bx+C}{x^2+9}$$

$$\rightarrow 2 \quad 9x^2 = A(x^2+9) + x(Bx+C)$$
$$= (A+B)x^2 + Cx + 9$$

Setting coefficient equal

$$x^2: \quad A + B = \quad 9 \qquad A = {}^2/_9$$
$$x^1: \quad C = \quad 0 \qquad B = \ {}^{83}/_9$$
$$x^0: \quad 9A = \quad 2 \qquad C = 0$$

$$\rightarrow \int \frac{2+x^4}{x^3+9x} dx = \int \left[x + \frac{{}^2/_9}{x} \quad \frac{\frac{83x}{9}}{x^2+9} \right] dx$$

$$= \boxed{\frac{1}{2}x^2 + \frac{2}{9}ln|x| \quad \frac{83}{18}ln|x^2+9| + C}$$

Q45	Math 27
	Miscellaneous Substitution

<u>Question:</u>

Evaluate $\int \frac{1}{1+\sin x+\cos x} dx$

<u>Solution:</u>

$$\int \frac{dx}{1+\sin x+\cos x} = \int \frac{\frac{2dz}{1+z^2}}{1+\frac{2z}{1+z^2}+\frac{1-z^2}{1+z^2}}$$

$$\text{let} \quad z = tan\left(\frac{x}{2}\right)$$

$$dx = \frac{2dz}{1+z^2}$$

$$sinx = \frac{2z}{1+z^2}$$

$$cosx = \frac{1-z^2}{1+z^2}$$

$$= \int \frac{2dz}{1+z^2+2z+1-z^2}$$

$$= \int \frac{dz}{1+z}$$

$$= \boxed{\ln\left|1+\tan\left(\frac{x}{2}\right)\right| + C}$$

Q46	Math 27
	Miscellaneous Substitution

<u>Question:</u>

Evaluate $\int \frac{1}{\sin x+\tan x} dx$

<u>Solution:</u>

$$= \int \frac{dx}{\sin x+\tan x} = \int \frac{\frac{2dz}{1+z^2}}{\frac{2z}{1+z^2}+\frac{2z}{1-z^2}} \quad \textit{half angle substitution}$$

$$= \int \frac{\frac{2dz}{1+z^2}}{\frac{2z[(1-z^2)+(1+z^2)]}{1-z^4}}$$

$$= \int \frac{\frac{2dz}{1+z^2}}{\frac{4z}{1-z^4}} = \frac{1}{2}\int \frac{(1-z^2)}{z} dz$$

$$= \boxed{\frac{1}{2}\left(\ln\left|\tan\left(\frac{x}{2}\right)\right| \quad \frac{\tan^2\left(\frac{x}{2}\right)}{2}\right) + C}$$

Q47	Math 27
	Miscellaneous Substitution

Question:

Evaluate $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$

Solution:

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt{x}} = \int \frac{6u^5\,du}{u^2+u^3}$$

Let $u = \sqrt[6]{x}$
$u^6 = x$
$6u^5 du = dx$

$$= \int \frac{6u^5\,du}{u^2(1+u)}$$

$$= \int \frac{6u^3\,du}{(1+u)}$$

$$= 6\int \left(u^2 \quad u + \frac{1}{u+1}\right) du$$

$$= \boxed{6\left(\frac{\sqrt{x}}{3} \quad \frac{\sqrt[3]{x}}{2} + \sqrt[6]{x} \quad \ln\left|\sqrt[6]{x}+1\right|\right) + C}$$

* Long division:

$u + 1 \,\big)\, u^3$, quotient: $u^2 \quad u + 1$

$\underline{u^3 + u^2}$

u^2

$\underline{u^2 \quad u}$

u

$\underline{u + 1}$

1

Q48	Math 27
	Miscellaneous Substitution

Question:

Evaluate $\int \frac{2x^3}{5-\sqrt{x^2-2}}\,dx$

Solution:

$$\int \frac{2x^3}{5-\sqrt{x^2-2}}\,dx = 2\int \frac{(u^2+2)u\,du}{5-u}$$

Let $u = \sqrt{x^2 \quad 2}$
$u^2 = x^2 \quad 2$
$2u = 2x dx$

$$= 2\int \frac{u^3+2u}{5-u}\,du$$

$$= \quad 2\int \frac{u^3+2u}{u-5}\,du$$

$$= \quad 2\int \left(u^2 + 5u + 27 + \frac{135}{u-5}\right) du$$

$$= \boxed{\frac{2}{3}(x^2 \quad 2)^{3/2} \quad 5(x^2 \quad 2) \quad 54\sqrt{x^2 \quad 2} \quad 270\ln\left|\sqrt{x^2 \quad 2} \quad 5\right| + C}$$

Q49	Math 27
	Miscellaneous Substitution

Question:

Evaluate $\int \frac{1}{5-3\cos x} dx$

Solution:

$$\int \frac{1}{5-3\cos x} dx = \int \frac{\frac{2dz}{1+z^2}}{5-3\left(\frac{1-z^2}{1+z^2}\right)}$$

$$= \int \frac{\frac{2dz}{1+z^2}}{\frac{5(1+z^2)-3(1-z^2)}{1+z^2}}$$

$$= \int \frac{2dz}{5+5z^2-3+3z^2} = \int \frac{2dz}{2+8z^2}$$

$$= \int \frac{dz}{1+4z^2} = \frac{1}{2}\int \frac{2dz}{1+4z^2}$$

$$= \frac{1}{2} Arctan(2z) + C$$

$$= \boxed{\frac{1}{2} Arctan\left(2\,tan\left(\frac{x}{2}\right)\right) + C}$$

let $z = tan\left(\frac{x}{2}\right)$

$$dz = \frac{\sec^2(x/2)}{2}$$

$$dx = \frac{2dz}{1+\tan^2(x/2)}$$

$$dz = \frac{2dz}{1+z^2}$$

Q50	Math 27
	Improper Integrals

Question:

Determine if the integral is convergent or divergent: $\int_{-\infty}^{+\infty} \frac{1}{1+x^2} dx$

Solution:

$$\int_{-\infty}^{+\infty} \frac{dx}{1+x^2} = \int_{-\infty}^{0} \frac{dx}{1+x^2} + \int_{0}^{+\infty} \frac{dx}{1+x^2}$$

$$= \lim_{t\to-\infty} \int_t^0 \frac{dx}{1+x^2} + \lim_{t\to\infty} \int_0^t \frac{dx}{1+x^2}$$

$$= \lim_{t\to-\infty} \left[Arctanx\right]_t^0 + \lim_{t\to\infty} \left[Arctanx\right]_0^t$$

$$= \lim_{t\to-\infty} Arctant + \lim_{t\to\infty} Arctant$$

$$= \frac{\pi}{2} + \frac{\pi}{2}$$

$$= \pi \qquad \boxed{\therefore t\ \ e\ integral\ is\ convergent}$$

Q51	Math 27
	Improper Integrals

Question:

Determine if the integral is convergent or divergent: $\int_0^3 \frac{1}{x-1}\,dx$

Solution:

$\int_0^3 \frac{dx}{x-1} = \int_0^1 \frac{dx}{x-1} + \int_1^3 \frac{dx}{x-1}$

*for $\int_0^1 \frac{dx}{x-1}$,

$\int_0^1 \frac{dx}{x-1} = \lim_{t\to 1^-} \int_0^t \frac{dx}{x-1}$

$= \lim_{t\to 1^-} \left[\ln|x \quad 1|\right]_0^t$

$= \left(\lim_{t\to 1^-} (\ln|t \quad 1|) \quad 0\right)$

$= \quad \infty \quad , divergent$

* $\boxed{since\ \int_0^1 \frac{dx}{x-1}\ is\ divergent, \int_1^3 \frac{dx}{x-1}\ is\ also\ divergent}$

Q52	Math 27
	Improper Integrals

Question:

Determine if the integral is convergent or divergent: $\int_{-\infty}^{+\infty} \frac{x}{(1+x^2)^2}\,dx$

Solution:

$\int_{-\infty}^{+\infty} \frac{xdx}{(1+x^2)^2} = \lim_{a\to-\infty} \int_a^0 \frac{xdx}{(1+x^2)^2} + \lim_{b\to+\infty} \int_0^b \frac{xdx}{(1+x^2)^2}$

$= \lim_{a\to-\infty} \left[\quad \frac{1}{2(1+x^2)}\right]_a^0 + \lim_{b\to+\infty} \left[\quad \frac{1}{2(1+x^2)}\right]_0^b$

$= \lim_{a\to-\infty} \left[\quad \frac{1}{2} + \frac{1}{2(1+a^2)}\right] + \lim_{b\to+\infty} \left[\quad \frac{1}{2} + \frac{1}{2(1+b^2)}\right]$

$= \quad \frac{1}{2} + \frac{1}{2} = 0$; $\boxed{since\ t\ \ e\ limit\ exists, t\ \ e\ integral\ is\ convergent}$

Q53	Math 27
	Improper Integrals

<u>Question:</u>

Determine if the integral is convergent or divergent: $\int_{-\infty}^{0} x^2 e^x dx$

<u>Solution:</u>

$\int_{-\infty}^{0} x^2 e^x dx$

$\int_{-\infty}^{0} x^2 e^x dx = \lim_{a \to -\infty} \int_{a}^{0} x^2 e^x dx$

$\rightarrow \int x^2 e^x dx = x^2 e^x \quad \int 2x\, e^x dx$

Let	$u = x^2$ $\mathrm{du} = 2xdx$	$dv = \mathrm{e}^{\mathrm{x}}\, dx$ $v = \mathrm{e}^{\mathrm{x}}$
Let	$u = 2x$ $\mathrm{du} = 2dx$	$dv = \mathrm{e}^{\mathrm{x}}\, dx$ $v = \mathrm{e}^{\mathrm{x}}$

$= x^2 e^x \quad 2xe^x + \int 2e^x\, dx = x^2 e^x \quad 2xe^x + 2e^x$

$\rightarrow \lim_{a \to -\infty} [e^x(2 \quad 2x + x^2)]_{a}^{0} = \lim_{a \to -\infty} [2 \quad e^x(a^2 \quad 2a + 2)]$

$= 2 \quad \lim_{a \to -\infty} \left(\frac{a^2}{e^{-a}} \quad \frac{2a}{e^{-a}} + \frac{2}{e^{-a}} \right)$

$= 2 \quad \boxed{\therefore t\ \ e\ integral\ is\ convergent}$

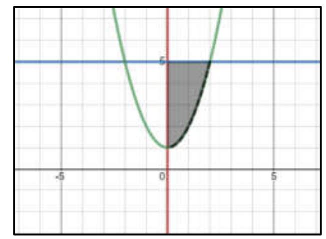
Q54	Math 27
	Area of a Plane Region

<u>Question:</u>

Find the first quadrant area bounded by the following curves: y=x^2+1, y=5 and x=0 (Use horizontal strips)

<u>Solution:</u>

Let $\quad u = y \quad 1$ $\mathrm{du} = dy$



$\rightarrow x = \sqrt{y \quad 1}$

$= \int_{1}^{5} (\sqrt{y \quad 1} \quad 0) dy$

$= \int_{1}^{5} \sqrt{u} du$

$= \left[\frac{u^{3/2}}{3/2} \right]_{1}^{5} = \left[\frac{(y-1)^{3/2}}{3/2} \right]_{1}^{5}$

$= \boxed{16/3\; units^2}$

Q55	Math 27
	Area of a Plane Region

Question:

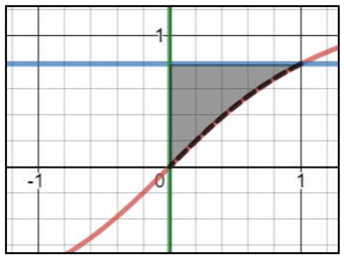
Find the first quadrant area bounded by the following curves: $y = \arctan x$, $y = \frac{\pi}{4}$ and $x = 0$

Solution:

$= \int_0^{\frac{\pi}{4}} tany dy$

$= [\ln|\sec y|]_0^{\frac{\pi}{4}}$

$= \boxed{5.3832 \times 10^{-3}\ units^2}$



Q56	Math 27
	Area of a Plane Region

Question:

Find the area between these two curves: $y = 9 \quad x^2$ and $2y = x^2 \quad 3x + 12$.

Solution:

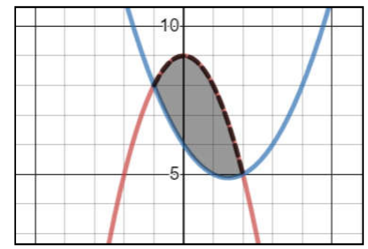
Solving for points of intersection:

$2y = x^2 \quad 3x + 12 \rightarrow \frac{1}{2}x^2 \quad \frac{3}{2}x + 6$

$\rightarrow 9 \quad x^2 = \frac{1}{2}x^2 \quad \frac{3}{2}x + 6$

$0 = \left(\frac{3}{2}x^2 \quad \frac{3}{2}x \quad 3\right)$

$0 = x^2 \quad x \quad 2 = (x \quad 2)(x + 1)\,;\ x = 2\,,1$



Solving for the Area using vertical strips

$A = \int_{-1}^{2} \left[(9 \quad x^2) \quad \left(\frac{1}{2}x^2 \quad \frac{3}{2}x + 6\right)\right] dx$

$= \int_{-1}^{2} \left(\quad \frac{3}{2}x^2 + \frac{3}{2}x + 3\right) dx$

$= \left[\quad \frac{3}{2}\left(\frac{x^3}{3}\right) + \frac{3}{2}\left(\frac{x^2}{2}\right) + 3x\right]_{-1}^{2}$

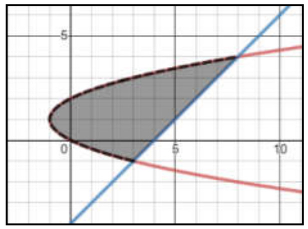
$= (\quad 4 + 3 + 6) \quad \left(\frac{5}{4} \quad 3\right)$

$= \boxed{\frac{27}{4}\ units^2}$

Q57	Math 27
	Area of a Plane Region

Question:

Find the area of the region bounded by these two curves: $x = y^2 \quad 2y$ and $x \quad y \quad 4 = 0$.



Solution:

Solving for points of intersection:

$\rightarrow y + 4 = y^2 \quad 2y$
$0 = y^2 \quad 3y \quad 4$
$0 = (y+1)(y \quad 4)\ ;\ y = \quad 1\ , 4$

Solving for the Area using horizontal strips

$$A = \int_{-1}^{4} [(y+4) \quad (y^2 \quad 2y)]dy$$

$$= \int_{-1}^{4} (\quad y^2 + 3y + 4)dy$$

$$= \left[\quad \frac{y^3}{3} + \frac{3y^2}{2} + 4y \right]_{-1}^{4}$$

$$= \left(\quad \frac{64}{3} + \frac{48}{2} + 16 \right) \quad \left(\frac{1}{3} + \frac{3}{2} \quad 4 \right)$$

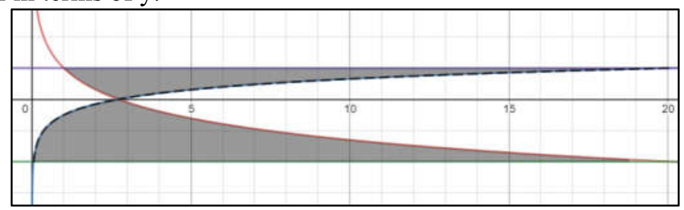
$$= 49 \quad \frac{130}{6} \quad \frac{9}{6}$$

$$= \boxed{\frac{125}{6}\ units^2}$$

Q58	Math 27
	Area of a Plane Region

Question:

Determine the area of the region bounded by the following curves: 1.) $x = e^{1+2y}$, 2.) $x = e^{1-y}$, 3.) y = -2, 4.) y = 1. Area of interest is below function 4, above function 3, above function 1 and below function 2. Only set up the integral in terms of y.



Solution:

Solving for points of intersection:

$$\rightarrow \frac{e^{1+2y}}{e^{1-y}} = 1 \rightarrow e^{3y} = 1 \rightarrow y = 0$$

Solving for the Area using horizontal strips

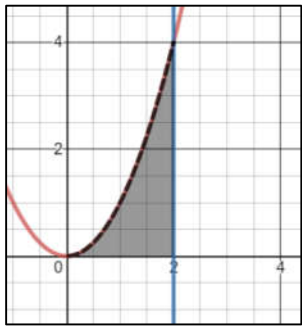
$$A = \boxed{\int_{-2}^{0} [(e^{1-y}) \quad (e^{1+2y})]dy + \int_{0}^{1} [(e^{1+2y}) \quad (e^{1-y})]dy}$$

Q59	Math 27
	Volume of a Solid of Revolution

Question:

Using disk method, find the volume of the region bounded by the curves: $y = x^2$, $x = 2$ and $y = 0$, revolved at $x = 2$. Only generate the integral using horizontal strips.

Solution:



→ $y = x^2$

→ $x = \sqrt{y}$

→ $r = 2 \quad \sqrt{y}$

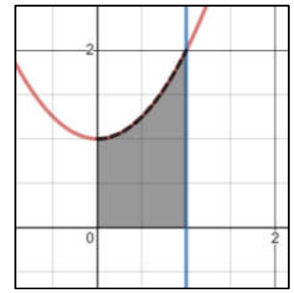
→ $\boxed{V = \pi \int_0^4 \left(2 \quad \sqrt{y}\right)^2 dy \; units^2}$

Q60	Math 27
	Volume of a Solid of Revolution

Question:

Using cylindrical shell method, find the volume of the region bounded by the curves: $y = x^2 + 1$ and $x = 1$, revolved at $y = 0$. Only generate the integral using horizontal strips.

Solution:



→ $x = \sqrt{y \quad 1}$

→ $\delta r = y$

→ $\delta \ _1 = 1$

→ $\delta \ _2 = 1 \quad \sqrt{y \quad 1}$

→ $\boxed{V = \pi \int_0^1 y dy + 2\pi \int_1^2 y\left(1 \quad \sqrt{y \quad 1}\right) dy \; units^2}$

Q61	Math 27
	Volume of a Solid of Revolution

Question:

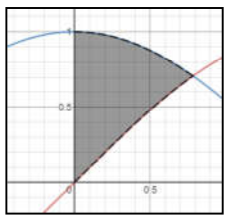
The region bounded by the y axis and the curves y = sin x and y = cos x for $0 \le x \le \frac{\pi}{4}$ is revolved about the x axis. Find the volume of the solid generated.

Solution:

$$\rightarrow V = \pi \int_0^{\frac{\pi}{4}} (\cos^2 x \quad \sin^2 x)dx = \pi \int_a^b \left[(f(x))^2 \quad (g(x))^2\right] dx$$

$$using\ identity\ \cos 2x = \cos^2 x \quad \sin^2 x$$

$$\rightarrow V = \pi \int_0^{\frac{\pi}{4}} \cos 2x\, dx$$



$$= \frac{1}{2}\pi[sin2x]_0^{\pi/4}$$

$$= \frac{1}{2}\pi\left(sin\frac{1}{2}\pi\right) = \frac{\pi}{2}$$

$$V = \boxed{\frac{\pi}{2}\ units^2}$$

Q62	Math 27
	Volume of a Solid of Revolution

Question:

Find the volume of the solid generated by revolving the region bounded by the curves $x = y^2 \quad 2$ and $x = 6 \quad y^2$ around their points of intersection (a vertical line). (Note: Only use the half of the region for computation)

Solution:

$$V = 4\pi \int_2^6 (x \quad 2)\sqrt{6 \quad x}dx$$

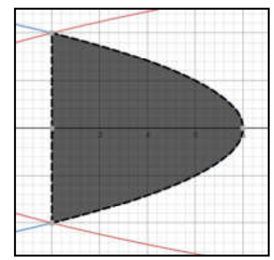
$$= 4\pi \int_2^0 (4 \quad u^2)u(2udu)$$

$$= 8\pi \int_2^0 (4u^2 \quad u^4)du$$

$$= 8\pi \left[\frac{4}{3}u^3 \quad \frac{u^5}{5}\right]_0^2$$

$$= \boxed{\frac{512\pi}{15}\ units^2}$$

Let $u = \sqrt{6 \quad x}$
$x = 6 \quad u^2$
$dx = 2udu$
Changing the bounds
$\rightarrow w\ \ en\ x = 2, u = 2$
$x = 6, u = 0$



Q63	Math 27
	Volume of a Solid of Revolution

Question:

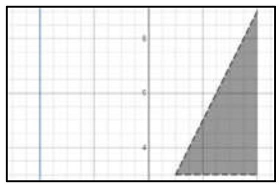
Use disk method to determine the volume of the solid generated by rotating the region bounded by y = 2x+1, x = 4 and y = 3, about the line x = -4. Use horizontal strips for showing the volume of the revolved area. Use horizontal strips.

Solution:

$$Inner\ radius = 4 + \frac{1}{2}(y \quad 1) = \frac{1}{2}y + \frac{7}{2}$$

$$Outer\ radius = 4 + 4 = 8$$

$$V = \boxed{\int_3^9 \pi \left[8^2 \quad \left(\frac{1}{2}y + \frac{7}{2}\right)^2\right] dy}$$



Q64	Math 27
	Centroid of a Plane Region

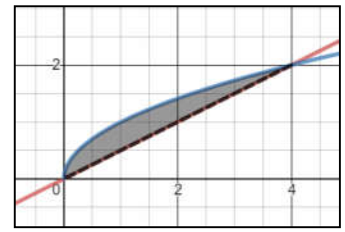
Question:

Find the centroid of the plane region bounded by the curves: $y = \sqrt{x}$ and $2y = x$, using vertical strips. Only generate the integral using vertical strips.

Solution:

$$\bar{x} = \boxed{\frac{\int_0^4 x\left(\sqrt{x} - \frac{x}{2}\right)dx}{\int_0^4 \left(\sqrt{x} - \frac{x}{2}\right)dx}}$$

$$\bar{y} = \boxed{\frac{\int_0^4 \frac{1}{2}\left(\sqrt{x} + \frac{x}{2}\right)\left(\sqrt{x} - \frac{x}{2}\right)dx}{\int_0^4 \left(\sqrt{x} - \frac{x}{2}\right)dx}}$$



Q65

Math 27

Centroid of a Plane Region

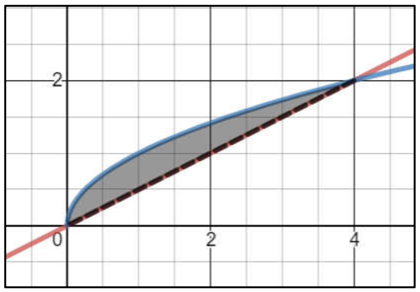
Question:

Find the integral yielding the centroid of the plane region of the example earlier using horizontal strips.

Solution:

$$\bar{x} = \boxed{\frac{\int_0^2 \frac{1}{2}(2y+y^2)(2y-y^2)dy}{\int_0^2 (2y+y^2)\mathrm{dy}}}$$

$$\bar{y} = \boxed{\frac{\int_0^2 y(2y-y^2)dy}{\int_0^2 (2y+y^2)\mathrm{dy}}}$$



Q66

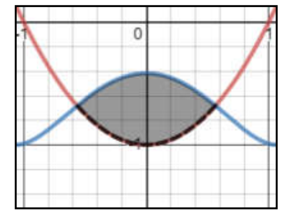
Math 27

Centroid of a Plane Region

Question:

Find the centroid of the region bounded by graph of $y = \cos(x^2 + 2)$ and $y = \; x^2 \quad 1$

Solution:



$\rightarrow \cos(x^2 + 2) = x^2 \quad 1$

$\rightarrow x = \pm 0.5651$

$\therefore \bar{x} = 0$

$\rightarrow A = 2\int_0^{0.5651}[\cos(x^2 + 2) \quad (x^2 \quad 1)]dx$

$= 0.4357\ units^2$

$\rightarrow M_x = \frac{1}{2}(2)\int_0^{0.5651}[\cos(x^2 + 2) \quad (x^2 \quad 1)]dx$

$= \quad 0.3969$

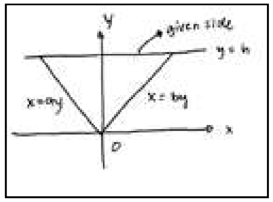
$\rightarrow \bar{y} = \frac{M_x}{A} = \frac{-0.3969}{0.4357} = \quad 0.7042$

$\therefore$ $\boxed{centroid\ is\ at\ (0, \quad 0.7042)}$

Q67	Math 27
	Centroid of a Plane Region

Question:

Prove that the distance from the centroid of a triangle to any side of the triangle is equal to one-third of the length of the height of that side.



Solution:

$$A = \int_0^h (a \quad b) y\, dy = \frac{1}{2}\left[(a \quad b) y^2\right]_0^h = \frac{1}{2}(a \quad b) \quad ^2$$

$$M_x = \int_0^h (a \quad b) y^2 dy = \frac{1}{3}\left[(a \quad b) y^3\right]_0^h = \frac{1}{3}(a \quad b) \quad ^3$$

$$\bar{y} = \frac{M_x}{A} = \frac{\frac{1}{3}(a-b)h^3}{\frac{1}{2}(a-b)h^2} = \frac{2}{3}$$

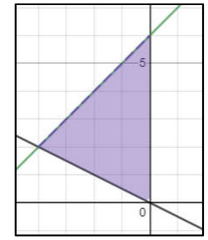
Solving for the distance between the centroid and the given side:

$$\rightarrow \quad \frac{2}{3} \quad = \boxed{\frac{1}{3}}$$

Q68	Math 27
	Centroid of a Plane Region

Question:

Find the center of mass for the triangle with vertices (0, 0), (-4, 2) and (0, 6). Only set up the integral.



Solution:

Finding the equation of triangle's edges:

(1) $y \quad (2) = \frac{0-(2)}{0-(-4)}\left(x \quad (x \quad 4)\right) = \quad \frac{1}{2}(x+4)$

$y = \quad \frac{1}{2}x$

(2) $y \quad (2) = \frac{6-(2)}{0+4}(x+4) = (x+4)$

$y = x + 6$

$$A = \int_{-4}^{0} \left[(x+6) \quad \left(\quad \frac{1}{2}x\right)\right] dx$$

$$M_x = \int_{-4}^{0} \frac{1}{2}\left[(x+6)^2 \quad \left(\frac{1}{2}x\right)^2\right] dx$$

$$M_y = \int_{-4}^{0} x\left[(x+6) \quad \left(\quad \frac{1}{2}x\right)\right] dx$$

$$\bar{x} = \frac{M_y}{A} = \frac{\int_{-4}^{0} x\left[(x+6)-\left(-\frac{1}{2}x\right)\right] dx}{\int_{-4}^{0}\left[(x+6)-\left(-\frac{1}{2}x\right)\right] dx}$$

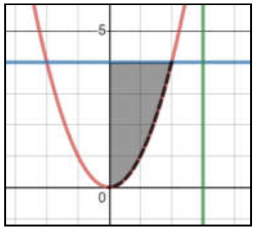
$$\bar{y} = \frac{M_x}{A} = \frac{\int_{-4}^{0} \frac{1}{2}\left[(x+6)^2-\left(\frac{1}{2}x\right)^2\right] dx}{\int_{-4}^{0}\left[(x+6)-\left(-\frac{1}{2}x\right)\right] dx}$$

Q69	Math 27
	Centroid of a Solid of Revolution

Question:

Set up an expression in terms of x that gives the centroid of a mass bounded by curves $y = x^2$, $y = 4$ and revolved around $x = 3$. Only set up the integral.

Solution:



$\rightarrow \delta \quad = 4 \quad x^2$

$\rightarrow \delta r = 3 \quad x$

$V = 2\pi \int_0^2 (3 \quad x)(4 \quad x^2)dx$

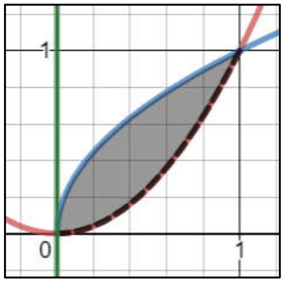
$$Centroid: (3, \bar{y}, 0)\ w\ \ ere\ \bar{y} = \frac{2\pi \int_0^2 \frac{1}{2}(4+x^2)(3-x)(4-x^2)dx}{2\pi \int_0^2 (3-x)(4-x^2)dx}$$

Q70	Math 27
	Centroid of a Solid of Revolution

Question:

Set up an expression in terms of y that gives the centroid of a mass bounded by curves $y = \sqrt{x}$, $y = x^2$ and revolved around the y axis.

Solution:



$Centroid: (\bar{x}, 0,0)\ w\ \ ere:$

$$\bar{x} = \frac{M_{yz}}{M} = \frac{2\pi \int_0^1 x[x(\sqrt{x}-x^2)]dx}{\int_0^1 x[x(\sqrt{x}-x^2)]dx}$$

Q71

Math 27

Centroid of a Solid of Revolution

Question:

Set up an expression in terms of x that gives the centroid of a mass bounded by curves $y = 5x^2$, $y = 4$ and revolved around $x = 3$.

Solution:

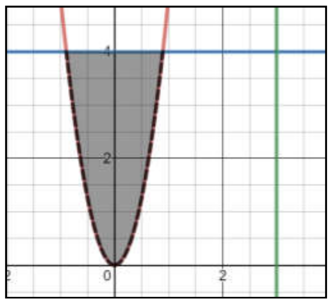
$\rightarrow A = \int_0^3 5x^2 dx = 45$

$for\ \bar{y}:\ 1215\pi = 45(2\pi\bar{y})$

$\bar{y} = \frac{1215\pi}{40(2\pi)} = 13.5$

$\bar{x} = 3$

$\boxed{\therefore\ Centroid\ is\ at\ (3,13.5,0)}$



Q72

Math 27

Centroid of a Solid of Revolution

Question:

Set up an expression in terms of x that gives the centroid of a mass bounded by curves $y = x^2$, $y = 4$ and revolved around $x = 3$. Use disk method. Only set up the integral.

Solution:

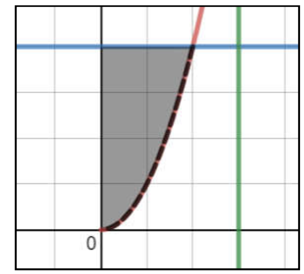
$r_o = 3$

$r_i = 3 \quad \sqrt{y}$

$V = \pi \int_0^4 \left(3^2 \quad (3 \quad \sqrt{y})^2\right) dy$

Centroid: $(3, \bar{y}, 0)\ w\ \ ere$

$\bar{y} = \frac{\boldsymbol{M_{xz}}}{M} = \frac{\pi \int_0^4 y(3^2-(3-\sqrt{y})^2)dy}{\pi \int_0^4 (3^2-(3-\sqrt{y})^2)dy}$



Q73	Math 27
	Length of an Arc of a Curve

Question:

Find the length of the curve $y = \frac{e^x + e^{-x}}{2}$ from x=0 to x=2.

Solution:

$$y = \frac{e^x + e^{-x}}{2} \qquad y' = \frac{e^x + e^{-x}}{2}$$

$$L = \int_0^2 \sqrt{1 + \left(\frac{e^x + e^{-x}}{2}\right)^2}\,dx = \int_0^2 \sqrt{1 + \frac{e^{2x} - 2 + e^{-2x}}{4}}\,dx$$

$$= \int_0^2 \sqrt{\frac{e^{2x} + 2 + e^{-2x}}{4}}\,dx$$

$$= \int_0^2 \sqrt{\left(\frac{e^x + e^{-x}}{2}\right)^2}\,dx = \int_0^2 \frac{e^x + e^{-x}}{2}\,dx$$

$$= \left[\frac{e^x - e^{-x}}{2}\right]_0^2$$

$$= \boxed{\frac{e^2 - e^{-2}}{2}}$$

Q74	Math 27
	Length of an Arc of a Curve

Question:

Find the length of the curve $y = e^x$ from x=0 to x=2. Only set up the integral.

Solution:

$$y = e^x$$

$$L = \int_a^b \sqrt{1 + (x')^2}\,dx$$

$$L = \boxed{\int_0^2 \sqrt{1 + e^{2x}}\,dx}$$

Q75	Math 27
	Length of an Arc of a Curve

Question:

Write down the integral which will compute the length of the part of the curve $y = \ln cosx$ from x = 0 to x = π/4.

Solution:

$$L = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}dx$$

$$\frac{d}{dx}(\ln|\cos x|) = \frac{1}{\cos x}(\quad \sin x)$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{1 + \left(\frac{-\sin x}{\cos x}\right)^2}\, dx$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2 x}\, dx$$

$$= \boxed{\int_0^{\frac{\pi}{4}} \sec x\, dx}$$

Q76	Math 27
	Length of an Arc of a Curve

Question:

Find the length of the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$, in the first quadrant from x=1 to x=a> 1.

Solution:

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1 \to y = b\left[1 \quad \left(\frac{x}{a}\right)^{2/3}\right]^{3/2}$$

$$\to y' = \frac{b}{a}\left[1 \quad \left(\frac{x}{a}\right)^{2/3}\right]^{1/2} \left(\frac{x}{a}\right)^{-1/3}$$

$$L = \int_{a/8}^{a} \sqrt{1 + \frac{b^2}{a^2}\left[1 \quad \left(\frac{x}{a}\right)^{2/3}\right]\left(\frac{x}{a}\right)^{-2/3}}\, dx$$

$$= \int_{a/8}^{a} \frac{1}{a}\left(\frac{x}{a}\right)^{-1/3} \sqrt{(a^2 \quad b^2)\left(\frac{x}{a}\right)^{2/3} + b^2 dx}$$

$$\to if\ b = a\ , = \int_{a/8}^{a} \left(\frac{x}{a}\right)^{-1/3} dx = \left[\frac{3a}{2}\left(\frac{x}{a}\right)^{2/3}\right]_{a/8}^{a} = \frac{9}{8}a$$

$$\to ot\ \ erwise, let\ u = (a^2 \quad b^2)\left(\frac{x}{a}\right)^{2/3}, du = \frac{2}{3}\left(\frac{a^2-b^2}{a}\right)\left(\frac{x}{a}\right)^{-1/3} dx$$

$$w\ \ en\ x = \frac{a}{8}, u = (a^2 \quad b^2)\left(\frac{1}{4}\right) + b^2 = \frac{1}{4}(a^2 + 3b^2)$$

$$w\ \ en\ x = a, u = (a^2 \quad b^2) + b^2 = a^2$$

$$\to Hence, L = \frac{3}{2(a^2-b^2)} \int_{\frac{a^2+3b^2}{4}}^{a^2} u^{1/2} du = \left[\frac{1}{a^2-b^2} u^{3/2}\right]_{\frac{a^2+3b^2}{4}}^{a^2}$$

$$\boxed{L = \frac{8a^3 - (a^2+3b^2)^{3/2}}{8(a^2-b^2)}}$$

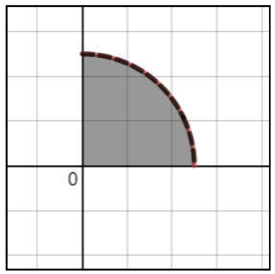
Q77	Math 27
	Area of a Surface of Revolution

Question:

Find the surface area of a sphere with a radius r.

Solution:

$$\rightarrow SA = \int_a^b 2\pi f(x)\sqrt{1+\left(f'(x)\right)^2}\,dx$$



$$= 2\pi \int_0^r \sqrt{r^2 \quad x^2}\sqrt{1+\left(\frac{-x}{\sqrt{r^2-x^2}}\right)^2}\,dx$$

$$= 2\pi \int_0^r \sqrt{r^2 \quad x^2}\sqrt{1+\frac{x^2}{r^2-x^2}}\,dx$$

$$= 2\pi \int_0^r \sqrt{(r^2 \quad x^2)+\frac{(r^2-x^2)x^2}{r^2-x^2}}\,dx$$

$$= 2\pi \int_0^r \sqrt{r^2 \quad x^2 + x^2}\,dx$$

$$= 2\pi \int_0^r r\,dx$$

$$= 2\pi r[x]_0^r = 2\pi r^2$$

$$W \quad ole\ sp \quad ere = 2SA = \boxed{4\pi r^2}$$

Q78	Math 27
	Area of a Surface of Revolution

Question:

Set up the definite integral giving the surface area of the curve $y = \cos x$, from x=0 to $x = \frac{\pi}{3}$, revolved at $x = 0$.

Solution:

Let $u = \sin x$
$du = cosx$

$$SA = \int_0^{\frac{\pi}{3}} 2\pi \cos x \sqrt{1+(\quad \sin x)^2}\,dx$$

Let $u = \tan\theta$
$du = \sec^2\theta\, d\theta$

$$= 2\pi \int_0^{\frac{\pi}{3}} \sqrt{1+u^2}\,du$$

$$= 2\pi \int_0^{Arctan\frac{\sqrt{3}}{2}} \sec^3\theta\, d\theta$$

$$= \pi \left[\frac{\sec\theta\tan\theta + \ln|\sec\theta+\tan\theta|}{2}\right]_0^{Arctan\frac{\sqrt{3}}{2}}$$

$$= \boxed{\pi\left(\frac{\sqrt{21}}{4} + \ln\left|\frac{\sqrt{7}+\sqrt{3}}{2}\right|\right)}$$

Q79	Math 27
	Area of a Surface of Revolution

Question:

Calculate the surface area of the surface defined by revolving the curve $y = \sqrt{9 \quad x^2}$ around the x-axis for $1 \le x \le 3$.

Solution:

$$\frac{d}{dx}\sqrt{9 \quad x^2} = \frac{1}{2\sqrt{9-x^2}}(\quad 2x) = \frac{-x}{\sqrt{9-x^2}}$$

$$SA = 2\pi \int_1^3 \sqrt{9 \quad x^2}\sqrt{1+\left(\frac{-x}{\sqrt{9-x^2}}\right)^2}\, dx$$

$$= 2\pi \int_1^3 \sqrt{9 \quad x^2}\sqrt{\frac{9}{9-x^2}}\, dx$$

$$= 2\pi \int_1^3 3dx$$

$$= \boxed{12\pi\ units^2}$$

Q80	Math 27
	Area of a Surface of Revolution

Question:

Find the area of the surface obtained by rotating the curve $y^2 = 4x + 4, 0 \le x \le 8$.

Solution:

$$y = 2\sqrt{x+1}$$

$$y' = \frac{1}{\sqrt{x+1}}$$

$$SA = \int_0^8 2\pi(2)(x+1)^{1/2}\sqrt{1+\left(\frac{1}{\sqrt{x+1}}\right)^2}\, dx$$

$$= 4\pi \int_0^8 (x+1)^{1/2}\sqrt{1+\frac{1}{x+1}}\, dx$$

$$= 4\pi \int_0^8 \sqrt{x+2}\, dx$$

$$= 4\pi \left[\frac{(x+2)^{3/2}}{3/2}\right]_0^8$$

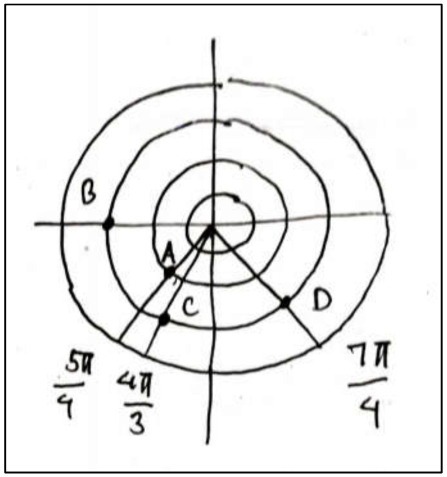
$$= \boxed{\frac{16\pi}{3}(3\sqrt{5} \quad 1)\sqrt{2}\ units^2}$$

Q81-84	Math 27
	Points in Polar Coordinates

Question:

Using polar coordinate plane, plot the following points:
A(2,-$\frac{5\pi}{4}$), B(3,99π), C(3,-$\frac{2\pi}{3}$), D(-4,-$\frac{3\pi}{4}$)

Solution:



Q85	Math 27
	Relationship Between the Cartesian and Polar System

Question:

Find the cartesian coordinate point of the polar point (10, $\frac{\pi}{4}$)

Solution:

$(r,\theta) = (10,\frac{\pi}{4})$

$x = 10\cos\frac{\pi}{4} = 5\sqrt{2}$

$y = 10\sin\frac{\pi}{4} = 5\sqrt{2}$

$\boxed{C(5\sqrt{2}, 5\sqrt{2})}$

Q86	Math 27
	Relationship Between the Cartesian and Polar System

Question:

Find the polar coordinate point of the cartesian coordinate point $(\sqrt{3}, 1)$, such that $r < 0$ and $0 \leq \theta < 2\pi$.

Solution:

$$C(\sqrt{3}, 1) = P\left(\ \ 2, \frac{7\pi}{6}\right)$$

$$r = \sqrt{(\sqrt{3})^2 + 1^2} = 2$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{6}$$

$$but\ since\ r = \ \ 2, \theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

Q87	Math 27
	Relationship Between the Cartesian and Polar System

Question:

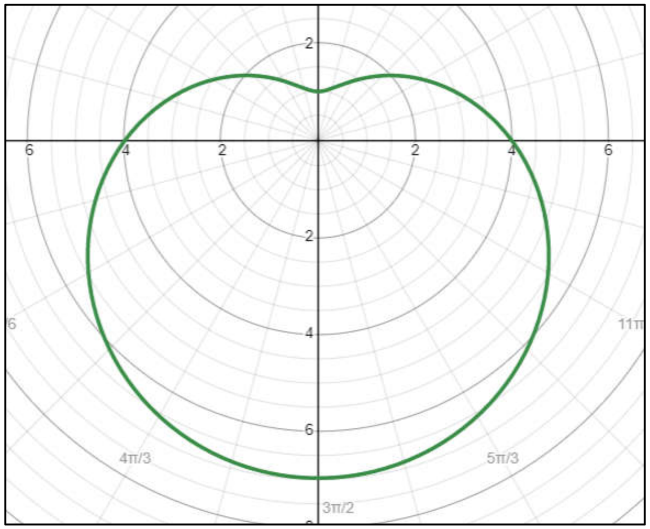
Find the cartesian coordinates of the polar point $(2,\ \ \frac{7\pi}{6})$

Solution:

$r = 4 \quad 3\sin\theta$

$\rightarrow \frac{a}{b} = \frac{4}{3} \in (1,2), \therefore\ dented$

Symmetry: $\frac{1}{2}\pi\ axis, points\ downward$



Q88	Math 27
	Relationship Between the Cartesian and Polar System

Question:

Find the polar coordinate point of the cartesian coordinate point $(3, \quad 3)$, such that $r > 0$ and $0 \leq \theta < 2\pi$.

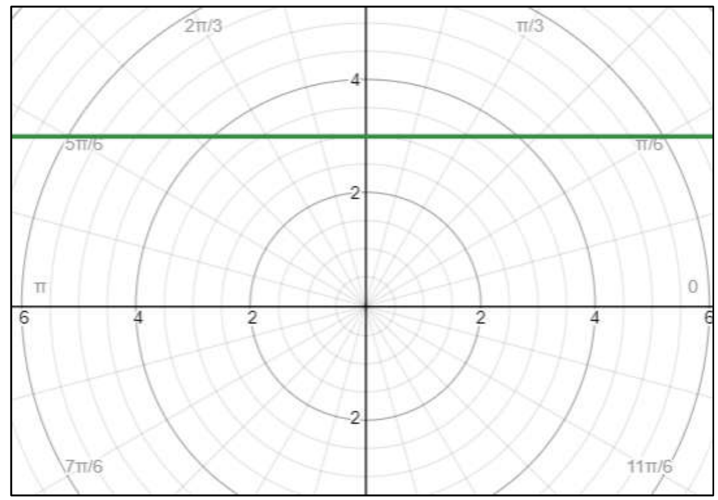
Solution:

Q89	Math 27
	Graphs of Polar Equations

Question:

Graph $r \sin \theta = \quad 3$

Solution:

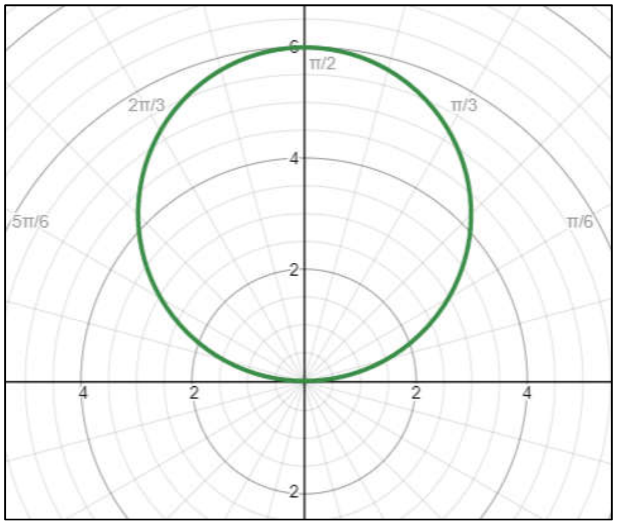


Q90	Math 27
	Graphs of Polar Equations

Question:

Graph $r = 3\sin\theta$

Solution:



Q91	Math 27
	Graphs of Polar Equations

Question:

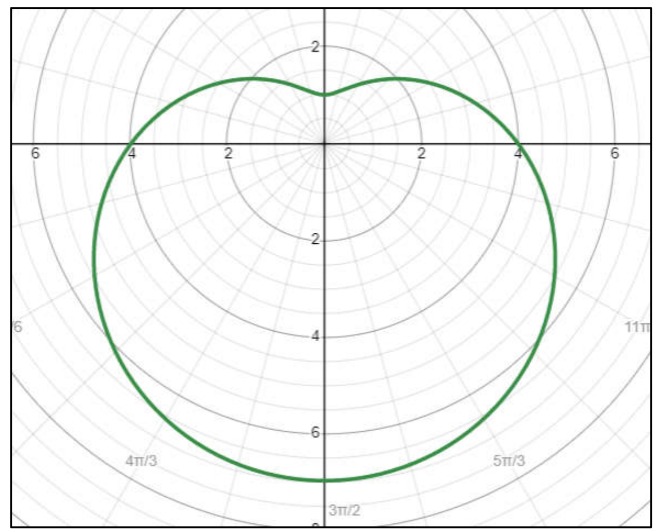
Given the polar equation, $r = 4 \quad 3\sin\theta$, determine the type of limacon, the symmetry and the direction it points. Also, plot the graph.

Solution:

$r = 4 \quad 3\sin\theta$

$\rightarrow \frac{a}{b} = \frac{4}{3} \in (1,2), \therefore\ dented$

Symmetry: $\frac{1}{2}\pi\ axis, points\ downward$

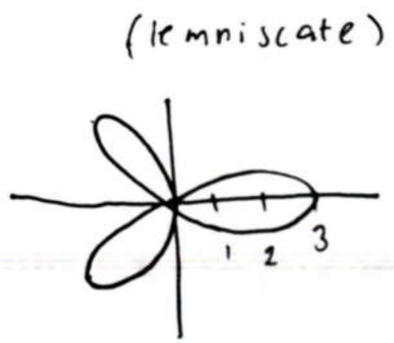


Q92	Math 27
	Graphs of Polar Equations

Question:

Graph $r^2 = 9 \sin 2\theta$

Solution:

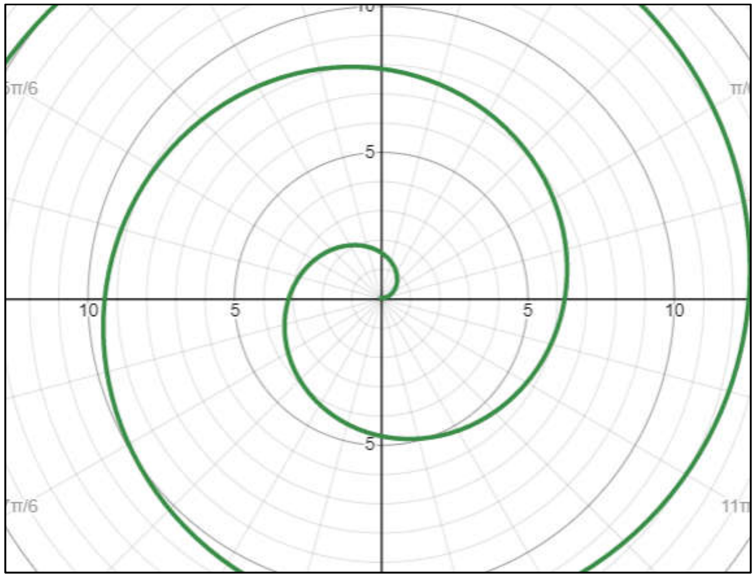


Q93	Math 27
	Graphs of Polar Equations

Question:

Graph r = θ

Solution:

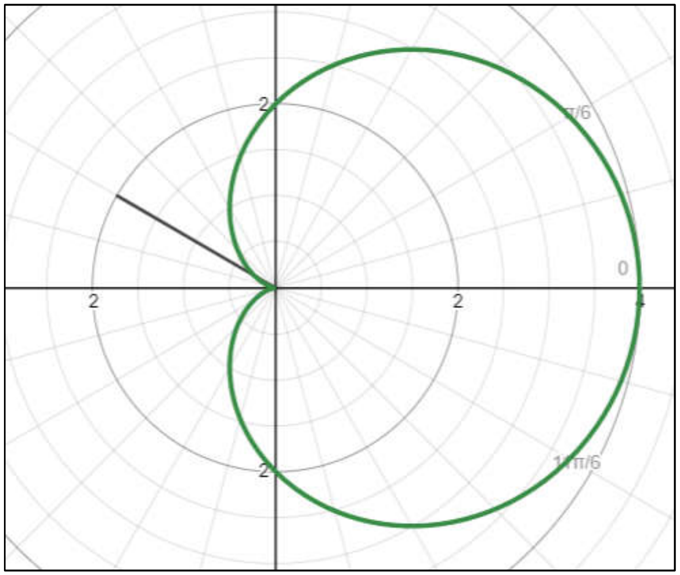


Q94	Math 27
	Graphs of Polar Equations

Question:

Graph $r = 2 + 2\cos\theta$

Solution:



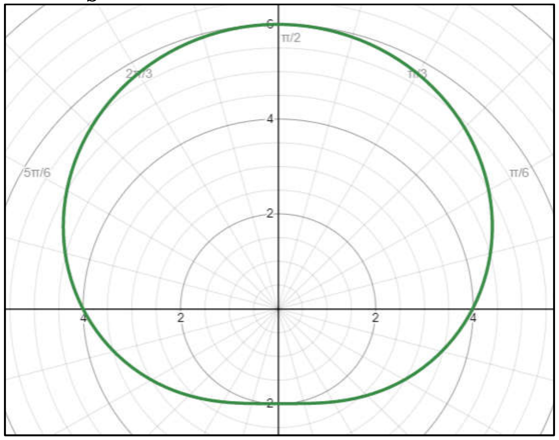
Q95	Math 27
	Graphs of Polar Equations

Question:

Graph $r = 4 + 2\sin\theta$

Solution:

$$\frac{a}{b} = 2; convex\ limacon, symmetric\ w/\ respect\ to\ \frac{\pi}{2}\ axis$$



θ	r
$\pi/2$	2
0	4
$\pi/2$	6

Q96

Math 27

Graphs of Polar Equations

Question:

Graph $r = 3 + 4\sin\theta$

Solution:

$\rightarrow \frac{a}{b} = \frac{3}{4}, \therefore \; limacon\ wit \quad a\ loop$

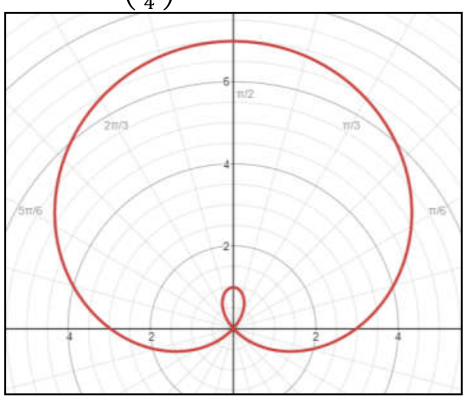
$Symmetry\ @\ \frac{\pi}{2} axis$

$\rightarrow find\ \theta\ t \quad at\ will\ give\ r = 0$

$0 = 3 + 4\sin\theta$

$\theta = \pi \qquad Arcsin\frac{-3}{4}$

$= Arcsin\left(\frac{-3}{4}\right)$



θ	r
$\pi/2$	-1
$\pi/6$	1
0	3
$\pi/6$	5
$\pi/2$	7

Q97

Math 27

Area in Polar Coordinates

Question:

Find the area of inside the cardiod, $r = 2 + 2\cos\theta$, but outside the circle with radius equal to 3. Only set up the integral.

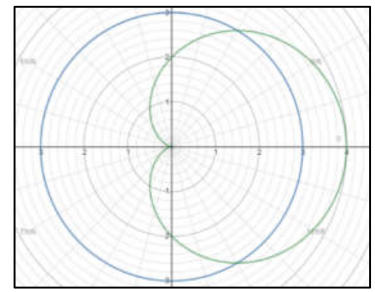
Solution:

$r = 3$

$r = 2 + 2\cos\theta$

$3 = 2 + 2\cos\theta$

$\theta = {}^{\pi}/_{3}\,, {}^{15\pi}/_{3}$



$A =$

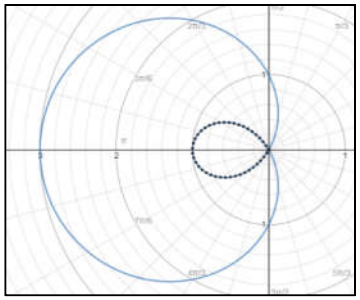
$$2\int_0^{\pi/3}((2 + 2\cos\theta)^2 \quad 3^2)d\theta$$

Q98	Math 27
	Area in Polar Coordinates

Question:

Set up the integral that will give the area inside the loop $r = 1 \quad 2\cos\theta$

Solution:



$r = 1 \quad 2\cos\theta$

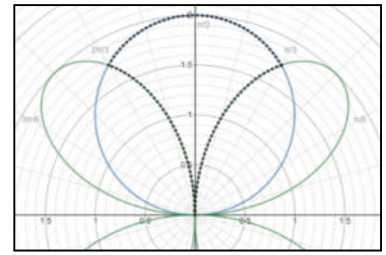
$$A = \boxed{\frac{1}{2}\int_{-\pi/3}^{\pi/3}(1 \quad 2\cos\theta)^2 d\theta}$$

Q99	Math 27
	Area in Polar Coordinates

Question:

Find the area of the region outside the rose ($r = 2\sin 2\theta$) and inside the circle ($r = 2\sin\theta$)

Solution:



area outside $r = 2sin2\theta$

area inside $r = 2sin\theta$

By symmetry,

$$A = 2\left(\frac{1}{2}\right)\int_{\pi/3}^{\pi/2}[(2\sin\theta)^2 \quad (2\sin 2\theta)^2]d\theta$$

$$= 2\int_{\pi/3}^{\pi/2}[(1 \quad \cos 2\theta) \quad (1 \quad \cos 4\theta)]d\theta$$

$$= 2\left[\frac{1}{4}\sin 4\theta \quad \frac{1}{2}\sin 2\theta\right]_{\pi/3}^{\pi/2}$$

$$= \frac{1}{2}\left(\sin 2\pi \quad \sin\frac{4}{3}\pi\right) \quad \left(\sin\pi \quad \sin\frac{2}{3}\pi\right)$$

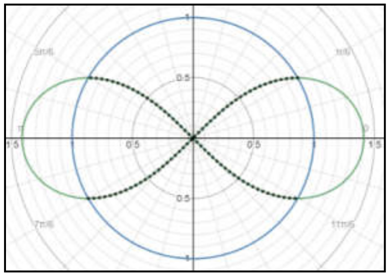
$$= \boxed{\frac{3}{4}\sqrt{3}\ square\ units}$$

Q100	Math 27
	Area in Polar Coordinates

Question:

Find the area of the region inside the lemniscate ($r^2 = 2\cos 2\theta$) and inside the circle ($r = 1$)

Solution:



Solving for intersections

let r = 1

$1 = 2cos2\theta$

$\theta = \frac{1}{6}\pi$

Let A_1 be the region between $\theta = 0$ and $\theta = \frac{1}{6}\pi$

$A_1 = \int_0^{\pi/6} d\theta = \frac{1}{12}\pi$

Let r = 0

$0 = 2cos2\theta$

$\theta = \frac{1}{4}\pi$

Let A_1 be the region between $\theta = \frac{\pi}{6}$ and $\theta = \frac{\pi}{4}$

$A_2 = \frac{1}{2}\int_{\pi/6}^{\pi/4}(2cos2\theta)d\theta$

$= \frac{1}{2}[sin2\theta]_{\pi/6}^{\pi/4}$

$= \frac{1}{2}\left(1 \quad \frac{\sqrt{3}}{2}\right)$

$\rightarrow A = 4\left[\frac{\pi}{12} + \left(\frac{1}{2}\left(1 \quad \frac{\sqrt{3}}{2}\right)\right)\right]$

$\boxed{A = \frac{\pi}{3} + 2 \quad \sqrt{3}}$